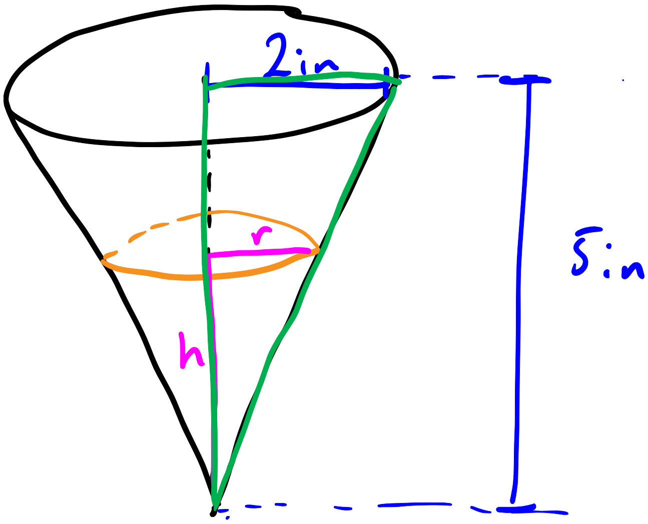
Ex: Coffee is draining out of a conical filter at a rate of $2.25\ \text{in}^3/\text{min}$. If the cone is 5 in tall and has a radius of 2 in, how fast is the coffee level dropping when the coffee is 3 in deep?



Equations:

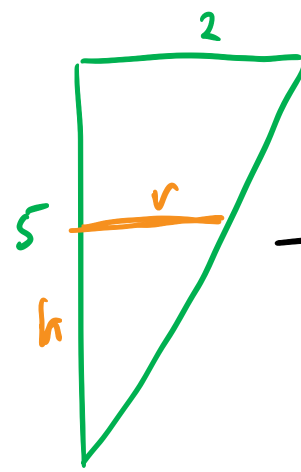
$$V = \frac{1}{3}\pi r^2 h$$

$$2h = 5r$$

Known: $\frac{dV}{dt} = -2.25$

$h = 3\text{in}$

Wanted: $\frac{dh}{dt} = ?$



$\rightarrow \frac{2}{5} = \frac{r}{h}$

$\hookrightarrow 2h = 5r$

$$\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{1}{3}\pi r^2 h\right)$$

want

$$\frac{dV}{dt} = \frac{1}{3}\pi\left(2r\frac{dr}{dt}h + r^2\frac{dh}{dt}\right)$$

missing!

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{2}{5}h\right)^2 h = \frac{4}{75}\pi h^3$$

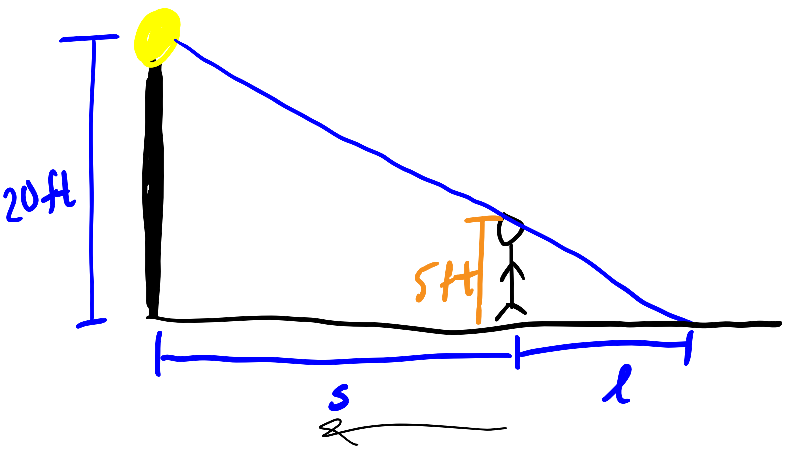
$$\left(r = \frac{2}{5}h\right)$$

$$\frac{d}{dt}\left(V = \frac{4}{75}\pi h^3\right) \longrightarrow \frac{dV}{dt} = \frac{4}{25}\pi h^2 \frac{dh}{dt}$$

$$-2.25\ \text{in}^3/\text{min} = \frac{4}{25}\pi (3\ \text{in})^2 \frac{dh}{dt}$$

$$\hookrightarrow \frac{dh}{dt} \approx -0.497\ \text{in}/\text{min}$$

Ex: A 5-ft-tall woman walks at 8 ft/s toward a streetlight that is 20 ft above the ground. What is the rate of change of the length of her shadow when she is 15 ft from the streetlight?



Equations:

$$\frac{5}{20} = \frac{l}{s+l} \to s = 3l$$

Known:

$$\frac{ds}{dt} = -8\,\frac{ft}{s}, \quad s = 15\ ft$$

Wanted: $\frac{dl}{dt}$

$$\frac{5}{20} = \frac{l}{s+l} \Rightarrow 5(s+l) = 20l \to 5s = 15l \to \underline{s = 3l}$$

$$\frac{d}{dt}(s = 3l) \to \frac{ds}{dt} = 3\frac{dl}{dt} \to \frac{dl}{dt} = \frac{1}{3}\frac{ds}{dt}$$

$$\to \frac{dl}{dt} = \left(-8\,\frac{ft}{s}\right)\cdot\frac{1}{3}$$

$$\to \frac{dl}{dt} = \frac{-8}{3}\ ft/sec.$$